AWADHESH PRATAP SINGH UNIVERSITY,

REWA (M.P.)

Ph. D. (Mathematics) Entrance Syllabus

2022-23 Onwards

AWADHESH PRATAP SINGH UNIVERSITY, REWA

Ph.D. (Mathematics) Entrance Syllabus form the session 2022-23 and onwards

Max. Marks : 100

Min. Pass Marks: 50

Note: Both the parts **A** and **B** shall consist of 50 objective type compulsory questions. Each question carries one mark.

Part A: Research Methodology

1. **Research Aptitude:** Research: Meaning, characteristics and types; Steps of research; Methods of research; Research Ethics; Paper, article, workshop, seminar, conference and symposium; Thesis writing: its characteristics and format.
2. **Reasoning (Including Mathematical):** Number series: Letter series; Codes; Relationships; Classification.
3. **Logical Reasoning:** Understanding the structure of arguments; Evaluating and distinguishing deductive and inductive reasoning; Verbal analogies: Word analogy-applied analogy; Verbal classification; Reasoning logical diagrams: Simple diagrammatic relationship, multi-diagrammatic relationship; Venn diagram: Analytical reasoning.
4. **Data Interpretation:** Sources, acquisition and interpretation of data; Quantitative and qualitative data; Graphical representation and mapping of data.
5. **Information and Communication Technology (ICT):** Meaning, advantages, disadvantages and uses; General abbreviations and technology; Basic of internet and e-mailing.

Part B: Mathematics

Unit-I Real Analysis and Linear Algebra

Real Analysis: Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limsup, liminf. Bolzano Weierstrass theorem, Heine Borel theorem. Continuity, uniform continuity, differentiability, mean value theorem. Sequences and series of functions, uniform convergence. Riemann sums and Riemann integral, Improper Integrals. Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral. Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation, inverse and implicit function theorems. Metric spaces, compactness, connectedness.

Linear Algebra: Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices, linear equations. Eigenvalues and eigenvectors, Cayley-Hamilton theorem. Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms.

UNIT – II Complex Analysis and Abstract Algebra

Complex Analysis: Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions. Analytic functions, Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem. Taylor series, Laurent series, calculus of residues. Conformal mappings, Mobius transformations.

Abstract Algebra: Permutations, combinations, pigeon-hole principle, inclusion-exclusion principle, derangements. Fundamental theorem of arithmetic, divisibility in Z, congruences, Chinese Remainder Theorem, Euler's Ø- function, primitive roots. Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems. Rings, ideals, prime and maximal ideals, quotient

rings, unique factorization domain, principal ideal domain, Euclidean domain. Polynomial rings and irreducibility criteria. Fields, finite fields, field extensions, Galois Theory.

UNIT – III Topology and Numerical Analysis

Topology: Basic concepts of topology, bases, subbases, subspace topology, order topology, product topology, quotient topology, metric topology, connectedness, compactness, countability and separation axioms, Urysohn's Lemma.

Numerical Analysis : Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

UNIT – IV ODEs and PDEs

Ordinary Differential Equations (ODEs): Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs. General theory of homogenous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function.

Partial Differential Equations (PDEs): Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs. Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

UNIT – V Functional Analysis and Operations Research

Functional Analysis: Normed linear spaces, Banach spaces, Quotient space of normed linear space and its completeness, Bounded linear operators and continuous operators on normed linear spaces, Finite dimensional normed linear spaces, Equivalent norms, Riesz Lemma, Bounded linear functionals, Dual spaces, Uniform boundedness principle, Open mapping and closed graph theorems, Hahn-Banach theorem for real & Complex linear spaces and some of its consequences, Reflexivity of Normed spaces, Inner product spaces, Convex sets, Riesz lemma on closed convex

set, Orthogonality and Orthonormality, Projection theorem, Bessel's inequality, Complete orthonormal sets, Riesz representation theorem, Reflexivity of Hilbert spaces, Self-adjoint operators, Projection, Normal and Unitary operators.

Operations Research: Linear programming models, convex sets, extreme points; Basic feasible solution, graphical method, simplex method, two phase methods, revised simplex method ; Infeasible and unbounded linear programming models, alternate optima; Duality theory, weak duality and strong duality; Balanced and unbalanced transportation problems, Initial basic feasible solution of balanced transportation problems (least cost method, north-west corner rule, Vogel's approximation method); Optimal solution, modified distribution method; Solving assignment problems, Hungarian method. Elementary queuing and inventory models. Steady-state solutions of Markovian queuing models.